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Introduction of

Core Burn-up Calculation with SRAC

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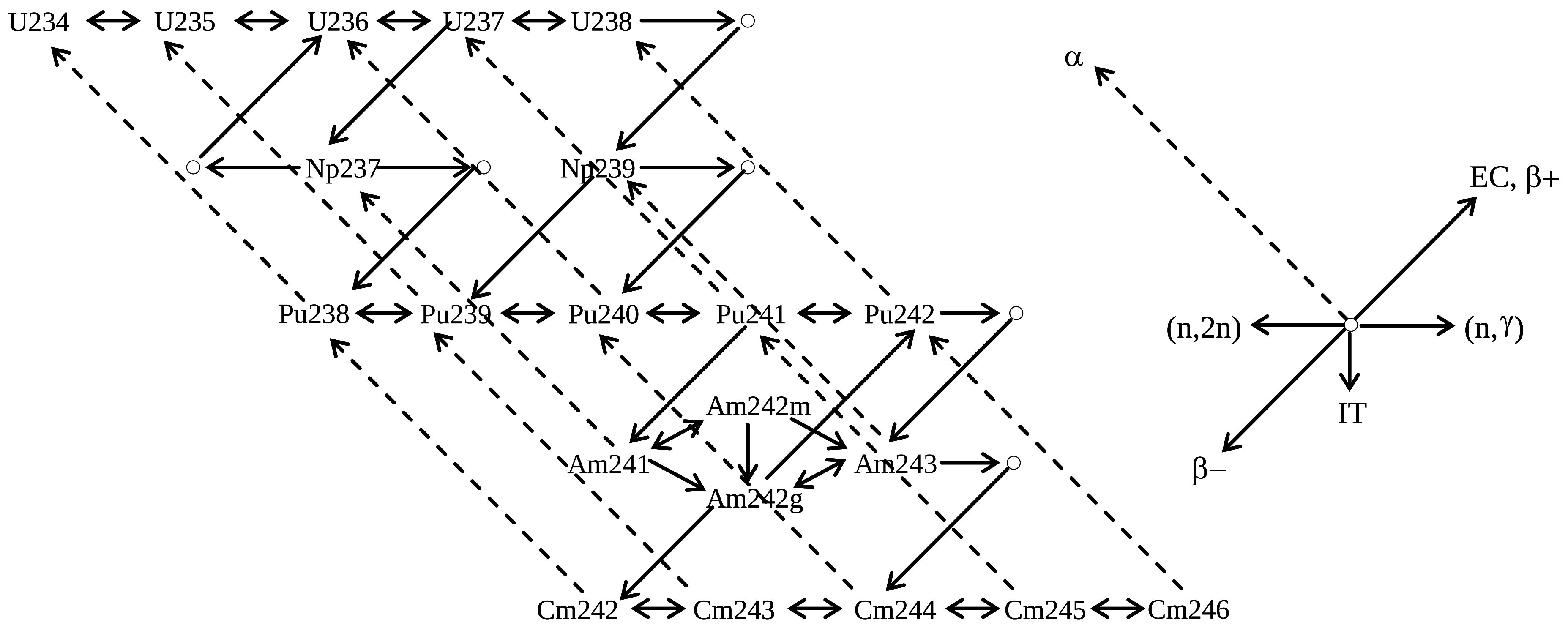
Available Burn-up Chain Models

Type of burn-up chain models (main purpose)	Name of chain model (=file name of burn-up chain data)	
	for thermal reactors	for fast reactors
Standard chain model (nuclear calculations)	u4cm6fp50bp16T th2cm6fp50bp16T	u4cm6fp50bp16F th2cm6fp50bp16F
General-purpose chain model (PIE analyses, etc.)	u4cm6fp104bp12T	
Detailed chain model (validation of other chain models)	th2cm6fp193bp6T	th2cm6fp193bp6F

Chain models are common for SRAC and MVP-BURN

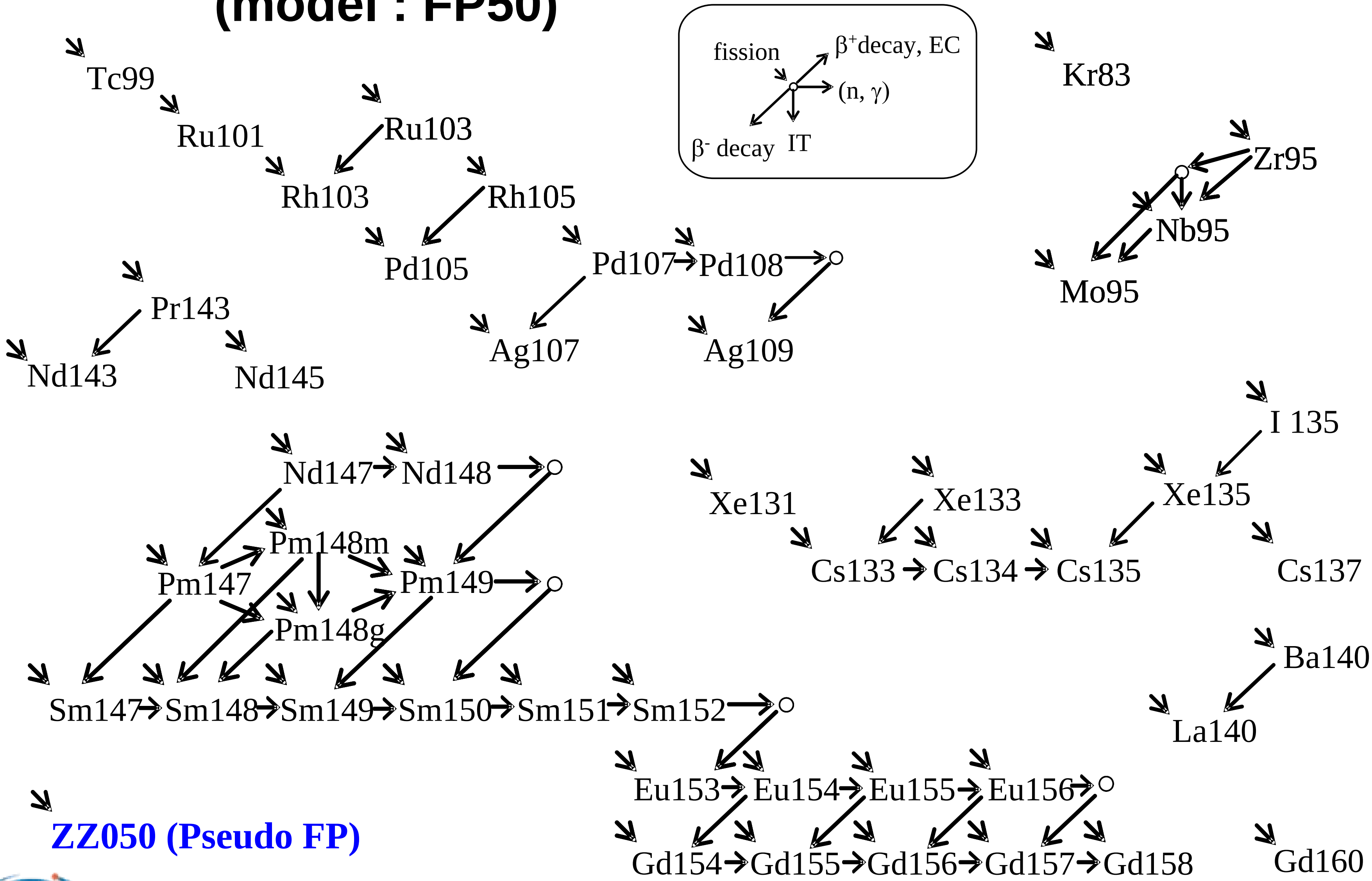
Chain Model for Heavy Nuclides



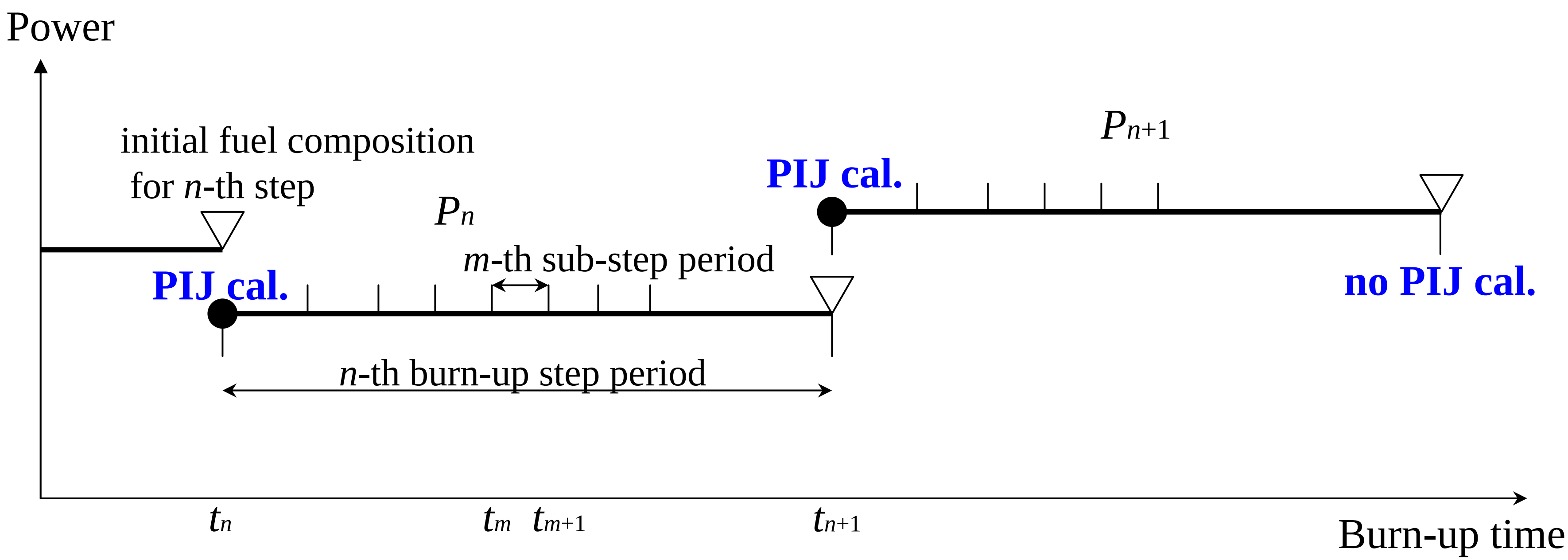
An example of burn-up chain model for actinides (model : U4CM6)

Chain Model for Fission Products

(model : FP50)

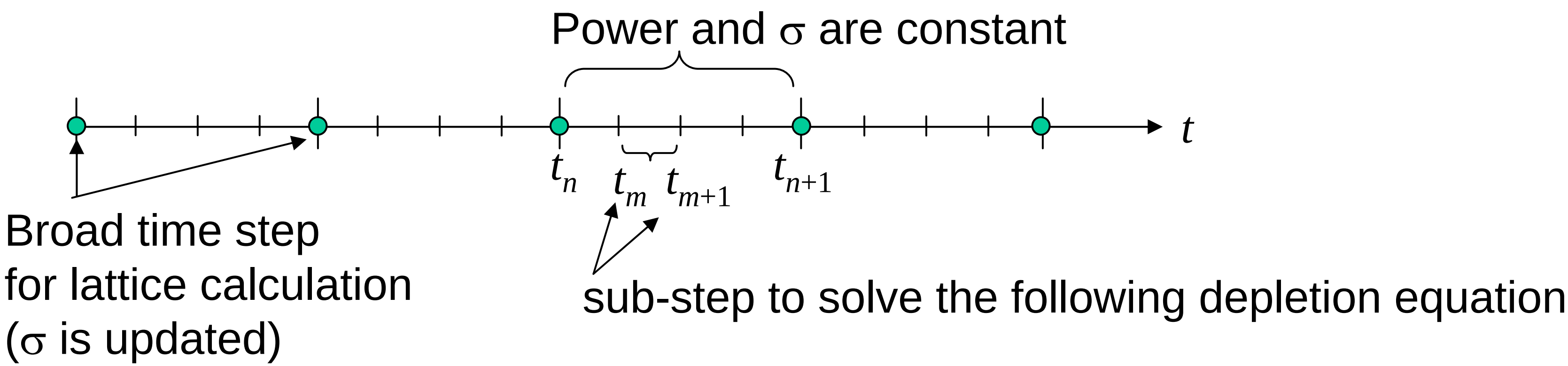


Time Steps during Burn-up



- **Power is assumed to be constant during a time step**
- **It is assumed that relative reaction rate does not change during a time step**

Depletion Equation



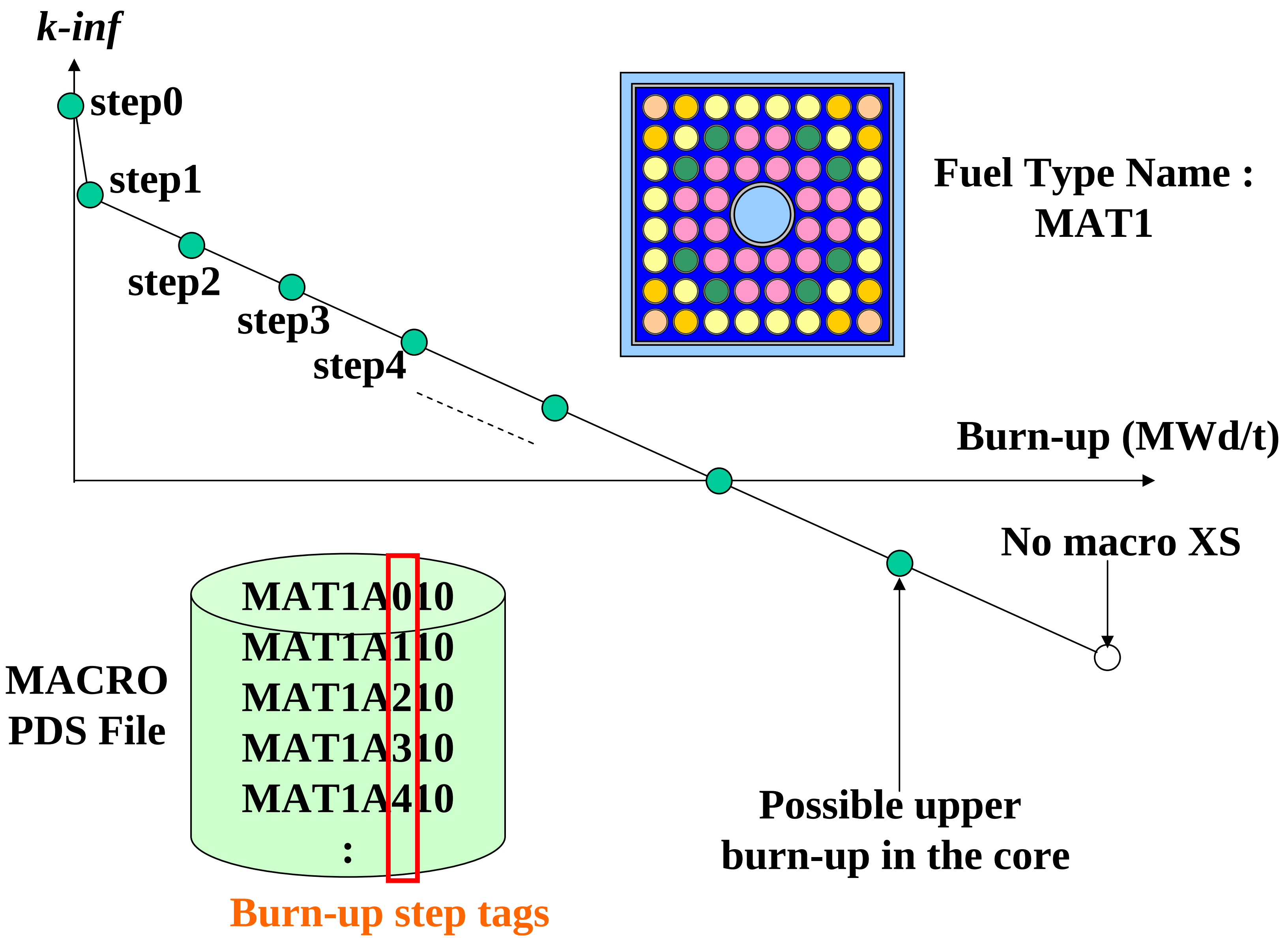
$$\frac{dN_i^z(t)}{dt} = \sum_{j\neq i} f_{j\to i}\lambda_j N_j^z(t) + \sum_{k\neq i} p_m\left\{g_{k\to i}C_k^z + \gamma_{k\to i}F_k^z + h_{k\to i}W_k^z\right\}N_k^z(t)$$
$$-\left[\lambda_i + p_m\left\{A_i^z + W_i^z\right\}\right]N_i^z(t)$$

⇒ Bateman’s method or Matrix exponential method ⇒ $N_i^z(t)$

where

i, j, k	:	depleting nuclide
z	:	burn-up zone
N	:	atomic number density
λ, f	:	decay constant and branch ratio
g, γ, h	:	yield fraction of each transmutation
F	:	microscopic fission reaction rate
A	:	microscopic absorption reaction rate
C	:	microscopic capture reaction rate
W	:	microscopic (n,2n) reaction rate
p_m	:	normalization factor to power (constant in each sub-step)

Cell Burn-up Calculation by SRAC



Macroscopic Cross-section Table

Burn-up Step	0	1	2	--------	10	11	--------
Burn-up(MWd/t)	0	100	500	--------	30000	35000	--------
Member Name	caseA0xp	caseA1xp	caseA2xp	--------	caseAAxp	caseABxp	--------
$\Sigma_{f,g}$							
$\nu\Sigma_{f,g}$							
$\Sigma_{a,g}$							
χ_g							
D_g							
:							

Rule of Burn-up-tag:

0,1,2,3,…9, (0~9 step)

A,B,C,….,Z, (10~35 step)

a,b,c,….z (36~61 step)

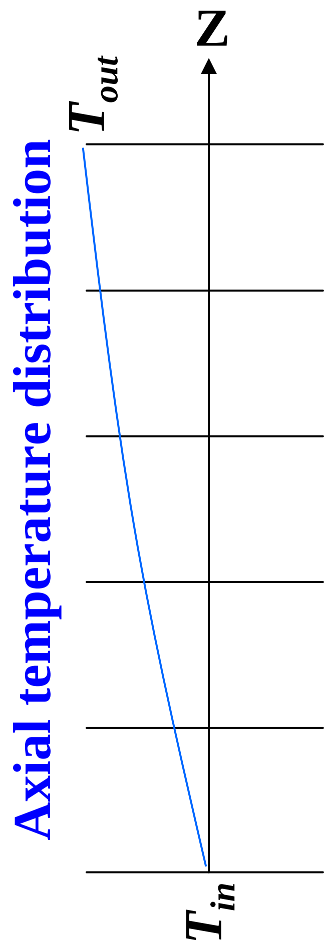
The Cygwin can not use these steps,
because file names with capital and small letters are not distinguished.

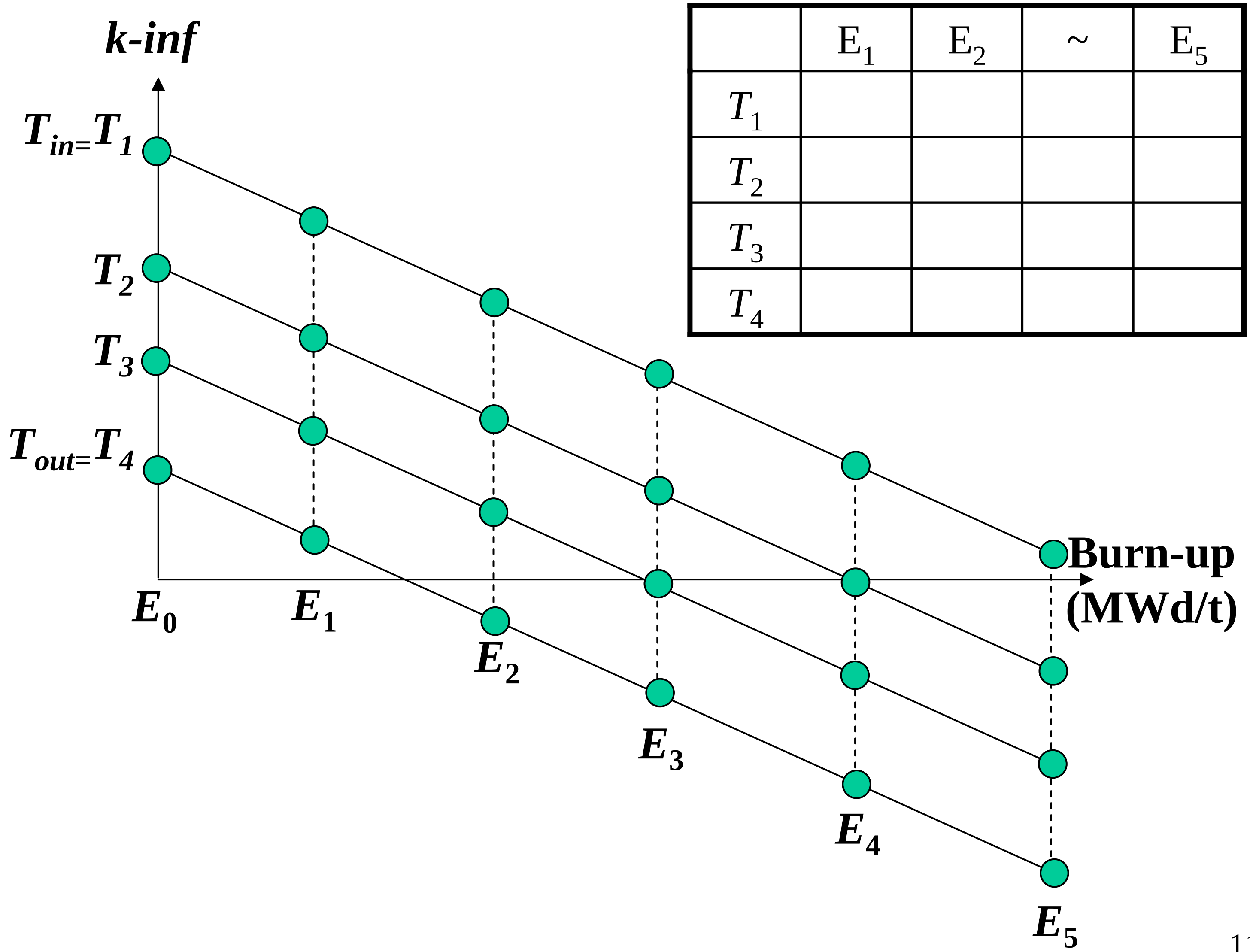
Atomic Number Density Table

Member Name: caseDNxT

Burn-up Step	0	1	2	--------	10	11	--------
Burn-up(MWd/t)	0	100	500	--------	30000	35000	--------
U-235 (n/cm^3)							
U-236							
U-238	Nuclides in the burn-up chain						
Pu-238							
:							

Multi-dimensional Cross-section Table





Two-dimensional table

	E_1	E_2	~	E_5
T_1				
T_2				
T_3				
T_4				

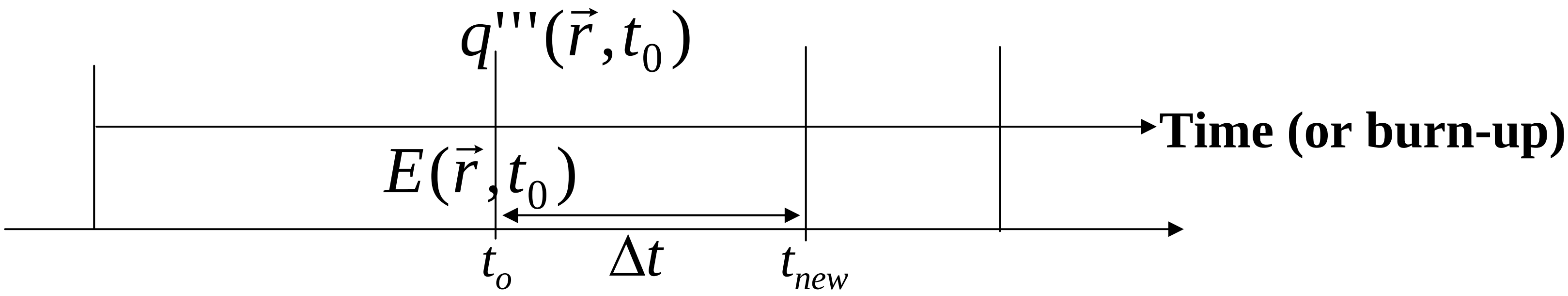
Functions of HIST Code

1. **Conversion of MACRO (PDS) file into the PS file**

2. **Registration and update of the history file**
   The history file keeps the following information:
   - Core geometry and material (i.e. XS-table)
   - Fuel element type (i.e. Fuel assembly type)
   - non-fuel element type (e.g. Control rods)
   - individual fuel element (burn-up, composition,
   - Operating record

3. **Print out information stored in the history file**
   (e.g. burn-up degree, atomic number densities, etc)

Functions of COREBN Code



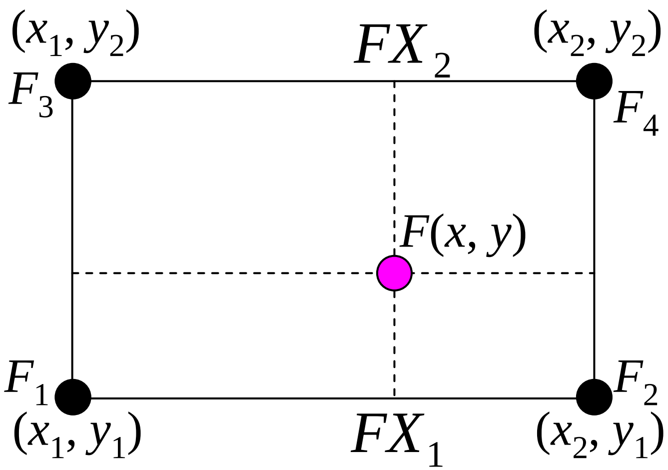
1. **Read necessary information from the history file:**
   Core geometry, loading positions of fuel / non-fuel elements, burn-up degree of each fuel elements
2. **Prepare macroscopic cross-sections by the interpolation on burn-up degree and other two parameters**
3. **Execute Diffusion calculation →** $\phi(\vec{r},t_0)$
4. **Power distribution →** $q'''(\vec{r},t_0) = c_1 \cdot \Sigma_f(\vec{r},t_0)\phi(\vec{r},t_0)$
5. **Update burn-up distribution**

next step (steps 2–5 repeat)

$$E(\vec{r},t_{new}) = E(\vec{r},t_0) + c_2 \cdot \int_{t_0}^{t_{new}} q'''(\vec{r},t_0)dt$$

$$\approx E(\vec{r},t_0) + c_2 \cdot q'''(\vec{r},t_0)\Delta t$$

Interpolation Scheme of Cross-sections



Two-dimensional interpolation on arbitrary two core parameters except for burn-up

$$R_x = \frac{x - x_1}{x_2 - x_1}, \quad R_y = \frac{y - y_1}{y_2 - y_1}$$

$$FX_1 = F_1 + (F_2 - F_1)R_x$$

$$FX_2 = F_3 + (F_4 - F_3)R_x$$

⇩

$$F = FX_1 + (FX_2 - FX_1)R_y$$

Finally, the value $F(x,y,z)$ is obtained by the liner interpolation between $F(x,y,z_1)$ and $F(x,y,z_2)$, where burn-up z lies between z_1 and z_2.

Xe-135 Correction

The COREBN has an option to correct absorption cross-sections upon assuming Xe-135 concentration is in equilibrium condition at each burn-up step. This correction is performed by

$$N_{Xe,\infty} = \frac{\left(y_{Xe} + y_I\right)\sum_g \left\{\Sigma_{f,g}\Phi_g\right\}}{\lambda_{Xe} + \sum_g \left\{\sigma_{Xe,g}\Phi_g\right\}}$$

$$\Sigma_{a,g} = \Sigma^0_{a,g} + \left(N_{Xe,\infty} - N^0_{Xe}\right)\sigma_{Xe,g} = \Sigma^0_{Xe,g} + \left[\frac{\left(y_{Xe} + y_I\right)\sum_g \left\{\Sigma_{f,g}\Phi_g\right\}}{\lambda_{Xe} + \sum_g \left\{\sigma_{Xe,g}\Phi_g\right\}} - N^0_{Xe}\right]\sigma_{Xe,g}$$

Local Xe-135 concentration in equilibrium

Interpolated absorption cross-section before correction

Interpolated Xe-135 concentration before correction

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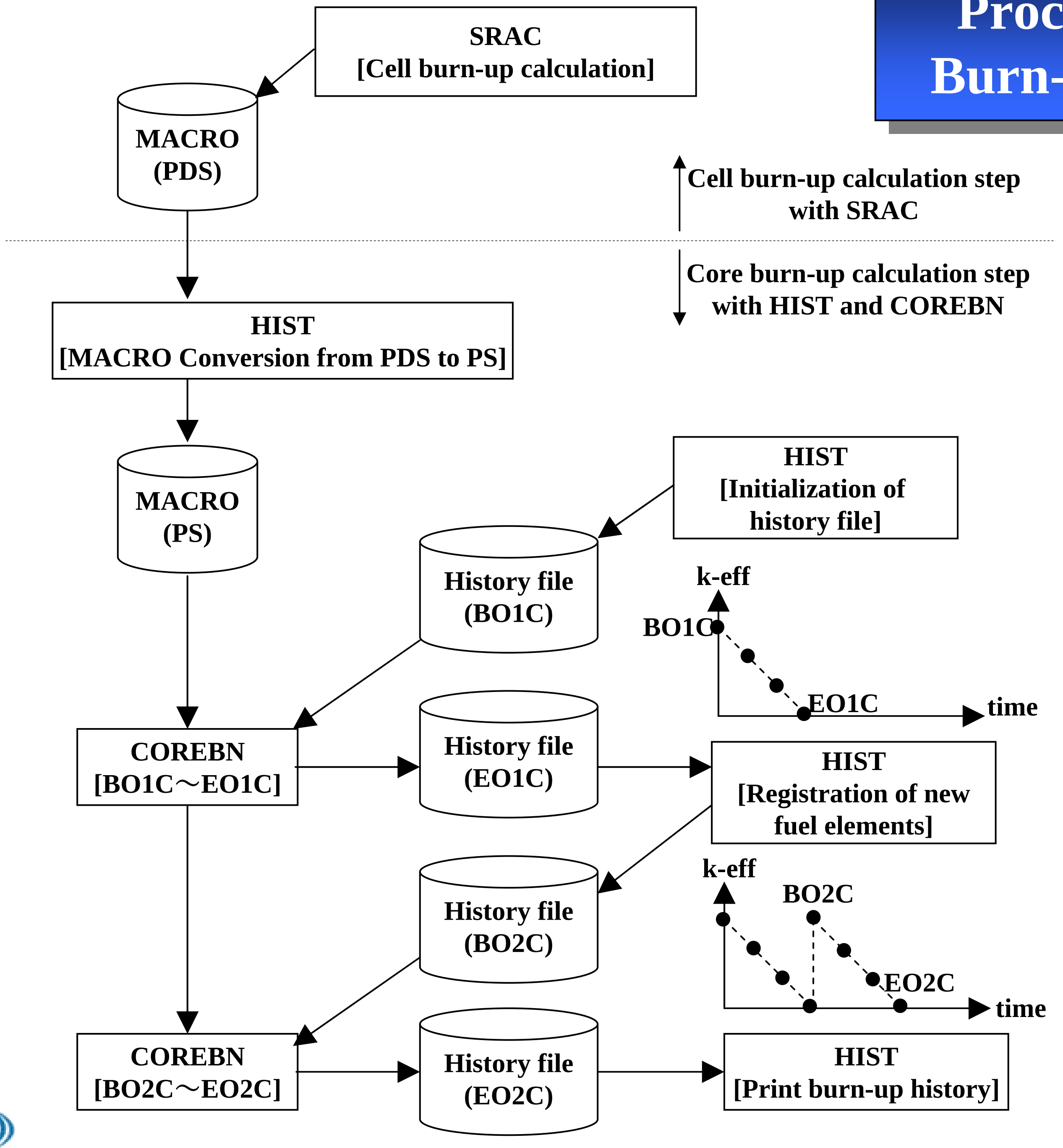
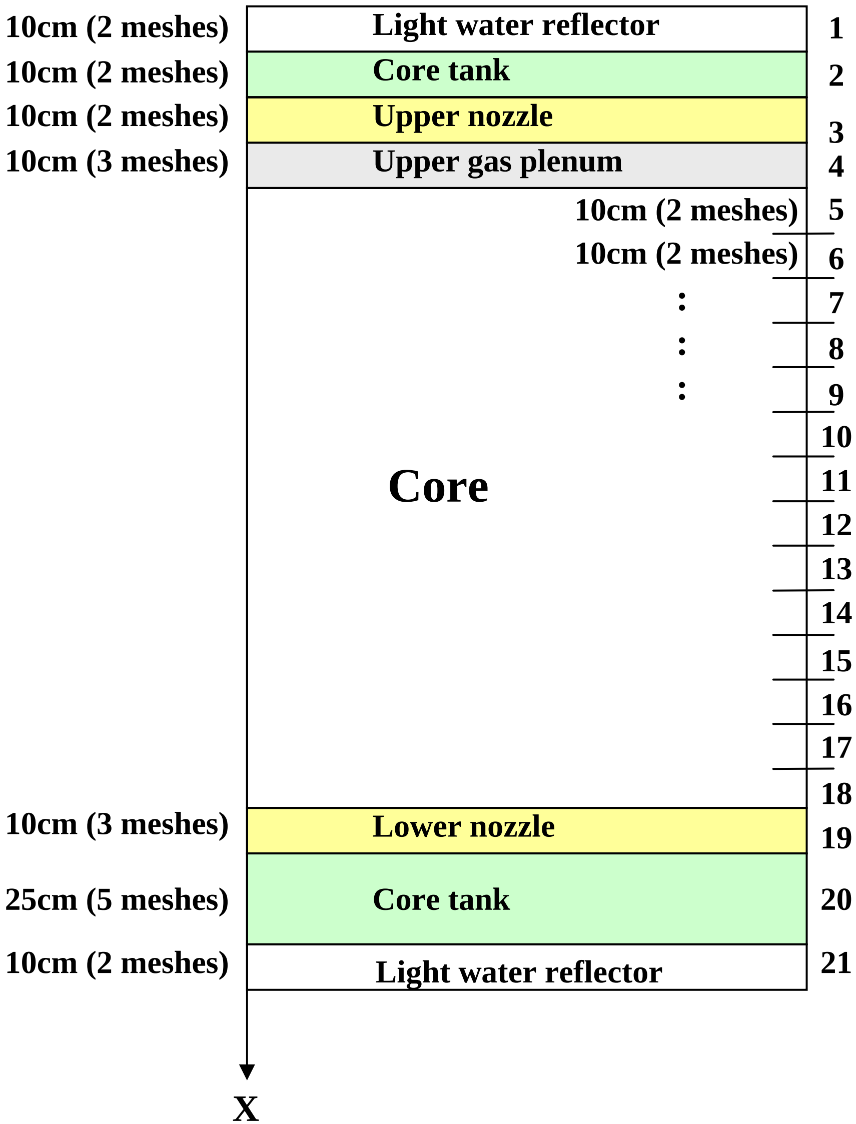
Procedure of Core Burn-up Calculation

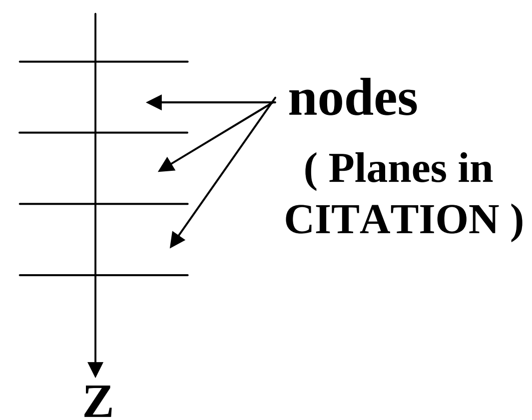


1-Dimensional Slab Core Problem (Sample-1)

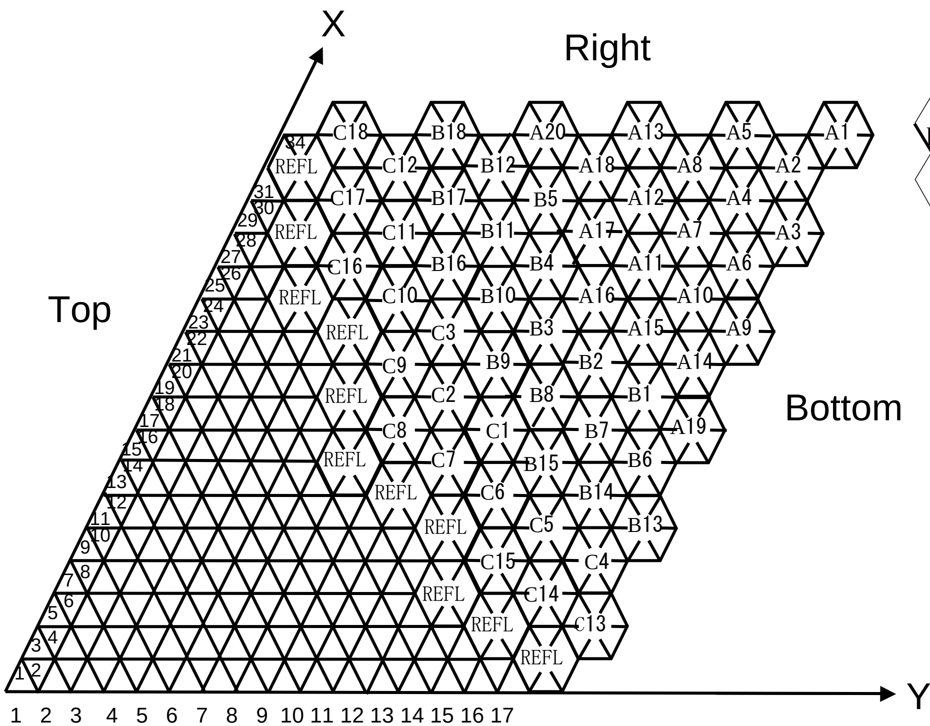


Division on X-direction can not be treated as node but fuel element

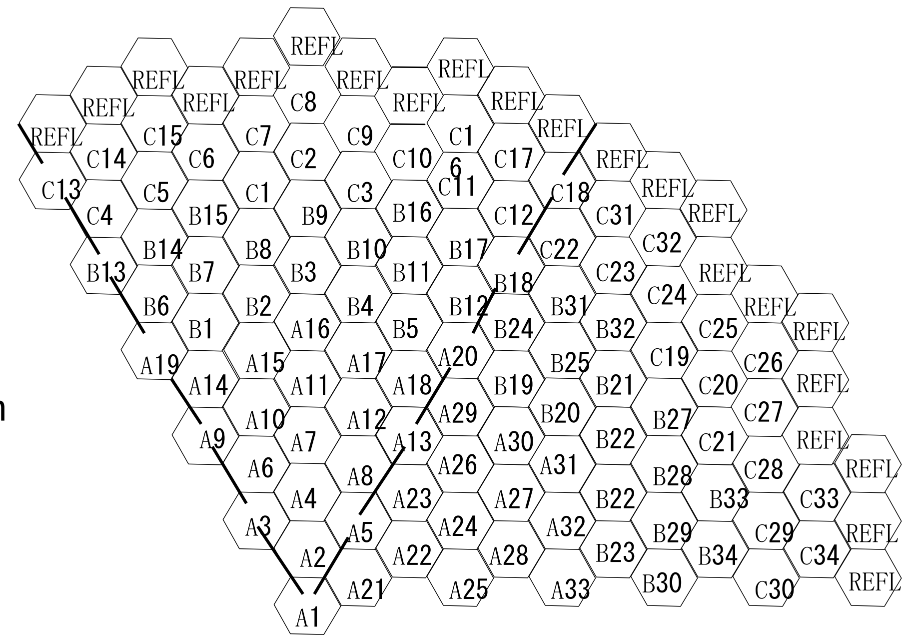
Fuel elements



2-Dimensional Triangular Mesh Problem (Sample-2)



Core calculation model



Refueling: C→B→A

registration of fuel elements

3-Dimensional Problem (Sample-3)